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ONE-DIMENSIONAL PROJECTION OF A LIQUID SHELL BY AN EXPLOSIVE CHARGE

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Introduction. The problem considered here is associated with the cavitational rupture of a liquid with a free surface under an explosive load. The concept of cavitational rupture is based on the fact that cleavages arise in cavitating liquids behind the leading edge of intense rarefaction waves in underwater explosions at shallow depths [1]. Detailed experimental analysis of the nature and dynamics of cavitation effects has shown that the rupture process has distinctive features and has a number of stages: a) unrestricted growth (a necessary condition) of cavitation nuclei up to the "bulk" bubble density, corresponding to a bulk concentration of 0.5-0.75; b) formation of foam-type structures and their breakup into fragments, i.e., cleavages; c) the transformation of the cavitating cleavages into a drop structure (structure of a splash dome on the free surface [1]).

Under an explosive load a real liquid, containing microinhomogeneities as cavitation nuclei being in essence a two-phase medium for rarefraction waves, is transformed into a gas-drop state during rupture. This process can be defined as the inversion of the two-phase nature of the medium and is a fundamental problem of explosion hydrodynamics, including a number of independent areas. One of them deals with the mechanism of the transformation of a foam structure into a drop structure. This is a sort of relaxation process, which calls for a detailed study. Getz and Kedrinskii [2] attempted to eliminate it in order to construct a model and analyze the dispersal of close packed drops and also proposed a model of instantaneous inversion of a cavitating liquid into a drop structure.

Another area involves numerical analysis of the parameters and structure of a cavitating liquid within the framework of the model of instantaneous relaxation of motion in the cavitation zone, making it possible to consider the dynamics of the zone up to high bulk concentrations [3]. As an example of this, we consider the problem of explosive projection of a liquid shell in the one-dimensional formulation.

Formulation of the Problem. A spherical explosive charge of initial radius r_0 lies at the center of a spherical liquid shell of radius r_1. After initiation of the charge at the center a detonation wave reaches the charge—shell contact boundary at time $t = 0$. The gas-kinetic flow formed at $t > 0$ is calculated for a wide range of $m = r_1/r_0$ ($m = 2$-10).

As the working medium we consider a real liquid, which is construed as a liquid with a natural content of microinhomogeneities of the type of microbubbles of free gas [4]. Their concentration is $\alpha_0 < 10^{-7}$. The effect of the compressibility of the gas component at such low values of α_0 is insignificant and so the propagation of shock waves in a real liquid is described well by the one-phase model. Taking this into account, we calculated the shock

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 90-96, November-December, 1992. Original article submitted September 18, 1991.

wave flow in the indicated region in the approximation of ideal compressibility of the liquid on the basis of [15]. The development of cavitation in the zone of discharge due to the interaction of the shock wave with the free surface of the real liquid is calculated with the model of instantaneous relaxation of the negative pressure in the unloading wave [3]. In this model the liquid is assumed to have zero strength because of the development of bubble cavitation in it.

The system of equations for calculating the propagation of a shock wave in water and the wave field in the noncavitating region of flow in mass Lagrange coordinates with the introduction of the artificial viscous pressure q has the form [5]

$$\partial r/\partial t = u; \tag{1}$$

$$\frac{\partial v}{\partial t} = \frac{v_0}{\xi^{\nu-1}}\frac{\partial (r^{\nu-1}u)}{\partial \xi}; \tag{2}$$

$$\frac{\partial u}{\partial t} = -v_0\left(\frac{r}{\xi}\right)^{\nu-1}\frac{\partial (p+q)}{\partial \xi}; \tag{3}$$

$$\frac{\partial \varepsilon}{\partial t} = -(p+q)\frac{\partial v}{\partial t}; \tag{4}$$

$$p = p(\varepsilon, v). \tag{5}$$

Here ν = 1, 2, 3, respectively, for plane, cylindrical, and spherical symmetry of flow; r and ξ are the Euler and Lagrange coordinates; t is the time; v_0 and v are initial and instantaneous specific volumes; p is the pressure; and ε is the specific internal energy. The quantity q was defined, in much the same way as in [5], as the sum of a linear and a quadratic term. We assumed that q = 0 in the region of the unloading wave, where $\partial\mu/\partial\xi > 0$.

For the detonation products we used an equation of state [6] of the form $p = (\gamma(\rho) - 1)\rho\varepsilon + \varphi(\rho)$ ($\gamma(\rho)$, $\varphi(\rho)$ are functions given in [6] and $\rho = 1/v$). For water we took the Walker–Sternberg equation of state [7], which applies for pressures up to $2.5 \cdot 10^4$ MPa:

$$p = f_1(\varepsilon)/v + f_2(\varepsilon)/v^3 + f_3(\varepsilon)/v^5 + f_4(\varepsilon)/v^7$$

($f_i(\varepsilon)$ (i = 1, ..., 4) are polynomial functions of the internal energy [7], which give a highly accurate approximation of data on the static and shock compressibility of water.

The cavitation process, in the approximation of instantaneous relaxation of the negative pressure, starts when zero pressure is reached in the wave. The pressure p in the liquid component of the mixture attains an equilibrium value equal to the vapor saturation pressure p_s (the elastic component relaxes instantaneously):

$$p = p_s \tag{6}$$

The flow of a cavitating liquid is described by virtually the same system of equations, with all the functions replaced by their average value (for the mixture) and the equation of conservation of momentum,

$$\partial u/\partial t = 0. \tag{7}$$

It follows from (7) that the profile of the mass velocity in the cavitation zone is "frozen": $u = F(\xi)$. The system is closed by the expression for the mean specific volume $\bar{v}$ of the mixture:

$$\bar{v} = v_1 + v_b. \tag{8}$$

Here v_1 is the specific volume of the liquid at the time of pressure relaxation, with $v_1 \neq v_0$ because of heating of the liquid in the shock wave; and v_b is the volume of bubbles per unit mass of mixture. We assume that the specific internal energy of the liquid is conserved ($\varepsilon = \varepsilon_1(\xi)$) and corresponds to its value at the time of pressure relaxation. If the condition corresponding to the disappearance of cavitation ($v_s = 0$ for $\partial\bar{v}/\partial t < 0$) occurs in the region under consideration, e.g., because of compression of the medium in the zone adjacent to the boundary with the detonation products, the calculation in the given subregion is continued with (1)-(5). The system of equations (1)-(8) is closed.

The problem is solved with the following initial and boundary conditions. At t = 0 when the detonation wave reaches the contact boundary the distribution of p, u, and v in the detonation products is found from analytical formulas given in [8]. The calculation was carried out for the explosive TG 50/50 with the specific volume $v_{exp} = 6.25 \cdot 10^{-4}$ m³/kg, the specific energy of the explosion $Q = 4.88 \cdot 10^3$ kJ/kg, and the Chapman–Jouguet parameters at the detonation wave from $p_j = 2.19 \cdot 10^4$ MPa, $v_j = 4.7 \cdot 10^{-4}$ m³/kg, and $u_j = 1.85 \cdot 10^3$ m/sec. The water ($r_0 < \xi < r_1$, where $\xi = r$ at t = 0) is under normal conditions: $p = p_0$, $v = v_0$, $u = 0$ (p_0 = 0.1 MPa, $v_0 = 10^{-3}$ m³/kg). The volume is constant ($p = p_0$) at the outer boundary of the liquid shell until the shock wave reaches it. After the shock wave interaction the pressure in the atmosphere is calculated as for an associated shock wave [9]:

$$D = \frac{\gamma_0 + 1}{4} u + \sqrt{\left(\frac{\gamma_0 + 1}{4}\right)^2 u^2 + c^2}, \quad p = p_0 + \frac{2}{\gamma_0 + 1} \frac{D^2}{v_g}\left(1 - \frac{c^2}{D^2}\right).$$

Here c = 330 m/sec is the velocity of sound in air; $\gamma_0 = 1.4$ is the adiabatic exponent of air; $v_g = 1.0$ m³/kg is its specific volume; and D is the shock wave velocity in air.

When solving the problem we check the balance of the integrated energy of the systems of detonation products + liquid shell. The total energy is written as the sum $e_s(t) = e_1(t) + e_2(t) + e_3(t) + e_4(t)$, where

$$e_1 = k \int\limits_{r_0(t)}^{r_1(t)} \frac{u^2}{2v} r^{\nu-1} dr, \quad e_2 = k \int\limits_{r_0(t)}^{r_1(t)} \frac{\varepsilon}{v} r^{\nu-1} dr$$

are the kinetic and internal energy of the liquid;

$$e_3 = k \int\limits_0^{r_0(t)} \frac{u^2}{2v} r^{\nu-1} dr, \quad e_4 = k \int\limits_0^{r_0(t)} \frac{\varepsilon}{v} r^{\nu-1} dr$$

are the kinetic and internal energy of the detonation products. Here $k = 4\pi, \pi, 1$ for ν = 3, 2, 1, respectively. We ignore the energy transferred to the atmosphere, because the shock wave is relatively weak in the air, and also ignore the energy associated with the vapor–gas phase in the bubbles, since its density and heat capacity are low in comparison with those of the liquid.

The numerical analysis is carried out in the dimensionless variables $r' = r/R$, $\xi' = \xi/R$, $t' = t/T$, $v' = v/V$, $\bar{v}' = \bar{v}/V$, $p' = p/P$, $u' = u/U$, and $\varepsilon' = \varepsilon/E$ (the upper-case letters denote dimensional constants). We take r_1, r_0, Q, v_0, and v_{exp} to be the determining parameters of the problem. We find the velocity U from the condition that the total energy of the shock wave is equal to the kinetic energy of the liquid shell projected like a solid: $U = (2Q/(m^\nu - 1)v_0 v_{exp})^{1/2}$. Substitution of dimensionless variables into the system of equations (1)-(8) shows that the flow remains similar when the parameters $T = R/U$, $P = U^2/V$, and $E = U^2$ are constant. This is confirmed by direct numerical intergration of the system (1)-(8) for various values of r_1 and r_0 at fixed values of $m = r_1/r_0$, Q, v_0, and v_{exp} with the initial and boundary conditions mentioned above. Henceforth we omit the primes after the dimensionless variables.

Results of Calculation. Figure 1 shows the results obtained from calculations of the dynamics of the distribution in the space of the pressure p, mass velocity u, and density 1/v ($1/\bar{v}$ in the cavitation zone) for times t = 0.19, 0.38, and 1.4 (lines 1-3, respectively). Here the vertical lines with a point correspond to inner boundary of the gas–liquid region and the lines without a point correspond to the outer boundary. We consider the case m = 2, $R = 3 \cdot 10^{-2}$ m, $\nu = 3$, $V = 10^{-3}$ m³/kg, which corresponds to the values U = 1494 m/sec, T = $2 \cdot 10^{-5}$ sec, and $P = 2.23 \cdot 10^3$ MPa. When the explosion has decayed at the contact boundary between the detonation products and the water at t = 0 a shock wave propagates in the liquid and, transforming, reaches the free surface (Fig. 1a-c, curves 1). The elastic energy of the liquid is transformed into kinetic energy in the unloading wave formed as a result of the shock wave interaction with the free surface. The rarefaction wave is focused on the explosion cavity and behind its front the mass velocity increases while the density of the two-phase mixture decreases as a result of the vigorous cavitation. The distribution of the pertinent parameters at the time corresponding to the approach of the unloading wave to the ex-

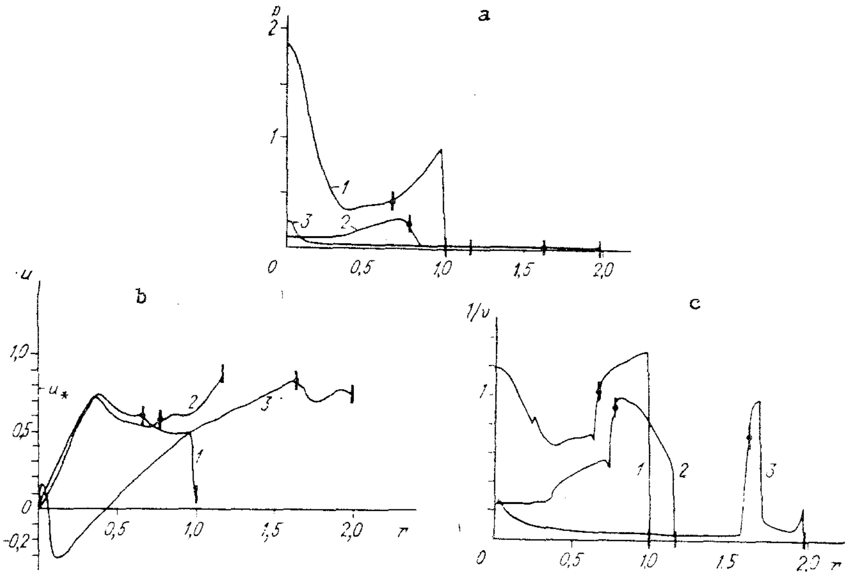

Fig. 1

plosion cavity is shown by curves 2 in Fig. 1a-c. We see that the interaction of the unloading wave with the detonation products causes the velocity of the contact boundary between the detonation products and the water to increase abruptly (Fig. 1b) and to remain high after that.

By the time t = 1.4 (Fig 1b, c) characteristic distributions of the velocity and density profile, with maxima at the boundaries, forms in the projected shell. The first maximum is determined by the increased pressure from the detonation products of the explosion, hindering the development of cavitation in a narrow layer that is adjacent to the detonation products and corresponds to a layer of homogeneous liquid. The two-phase undergoes inertial dilatation in the cavitation zone and the bulk concentration of the gas phase corresponding to the lower value of the close packing of bubbles ($\alpha_* = 0.5$) is already reached at $t_* \approx 0.6$ in the example under consideration. The counter pressure generated behind the associated shock wave in the air causes a local increase in the density of the gas—liquid mixture in the neighborhood of its outer boundary (Fig. 1c, curve 3). This increase in the density of the medium near the free surface can be treated as the formation of a "cleavage" [10, 11]. The distinctive features of the density variation in the projected shell are confirmed qualitatively by the experimental results of [12], where the fine structure of the flow was recorded by the dynamic pressure and showed that it is characterized by a zone of developing cavitation and a homogeneous layer of liquid on its inner boundary. A characteristic feature of the development of cavitation near a free surface is that the free surface maintains a high mass velocity which persists for a relatively long time (Fig. 1b, curves 2, 3). A secondary wave forms after reflection of the detonation products from the center. In the example under consideration this wave propagates through the detonation products at a lower velocity than that of the contact boundary between the detonation products and the water, which thus do not interact.

Figure 2 shows the graph of the balance of the integrated energy e_s as the time dependence of the sum of its individual components, relative to the total initial energy of the explosive e_0. The deviation of e_s/e_0 from 1 during the reading was less than 1%. The kink in the upper curve of Fig. 2 at t = 0.2 corresponds to the arrival of the shock wave at the free surface of the liquid and the onset of transformation of the elastic energy of the liquid into kinetic energy. The relative kinetic energy e_1/e_0 of the liquid increases with time because the explosion products do work to accelerate the liquid of the shell. The ratio e_1/e_0 tense to 0.6 with time. When this is taken into account the average velocities u_* of the thin liquid shells projected by the explosion can be estimated as $u_* = \sqrt{0.6}U$ (see Fig. 1b).

After relaxation of the pressure the relative internal energy e_2/e_0 of the liquid is the residual thermal energy due to heating of the liquid at the shock-wave front and cooling along the unloading isentrope. In the liquid, where the pressure at the shock-wave front exceeds $5 \cdot 10^3$ MPa, wet steam can form during unloading to 0.1 MPa [6] since the unloading isentrope intersects the saturation curve. The indicated region for $\nu = 3$ and 2 is bounded by the ranges $1 < \xi/r_0 < 1.25$ and $1 < \xi/r_0 < 1.4$, respectively.

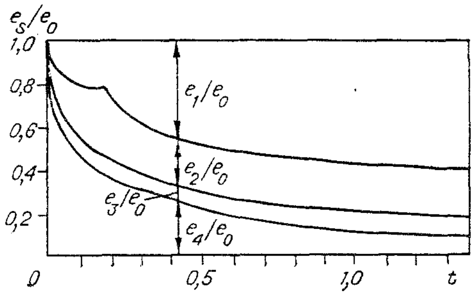

Fig. 2

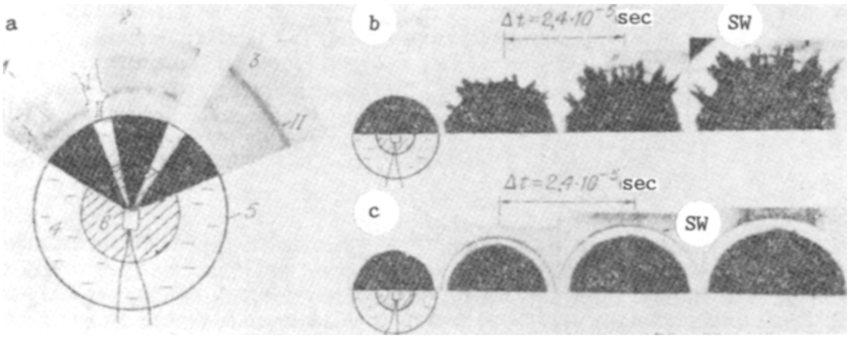

Fig. 3

Calculations showed that as $m = r_1/r_0$ increases the main stages of the process are qualitatively similar to those mentioned above. Secondary compression waves, reflected from the center and the contact boundary between the detonation products and water, may enter into multiple interactions, but these do not have any appreciable effect on the dynamics of the expanding shell. We merely note that with increasing m there is a natural decrease in the cavitation rate, characterized by $\partial\bar{v}/\partial t$ in the given model. This is attributed to a drop in the pressure gradient behind the front of the shock wave, reaching the free surface, and hence a decrease in the strain (tension) rate of the medium in the cavitation zone. As for the symmetry of the process, in real time the density of the mixture in the cavitation zone at a fixed specific explosion energy decreases more rapidly for a sphere than for a cylinder.

Experiment. The structure of the liquid shell projected by the explosion was recorded experimentally by taking pulsed x-ray photographs. Figure 3a shows a diagram of the experimental arrangement and successive x-ray photographs 1-3 of parts of it, taken at $t = 29\cdot10^{-6}$, $34\cdot10^{-6}$, and $53\cdot10^{-6}$ sec, respectively. The liquid (water) 4 was bounded externally by a rigid spherical shell 5 of inner radius $r_1 = 0.03$ m and thickness 10^{-3} m, made of epoxy-impregnated gauze. A TG 50/50 charge 6 of radius $r_0 = 0.015$ m was set off by an LD-34 detonator. As in the calculation given above, m = 2. The x-ray photographs were obtained by double exposure: freeze-frame (dark segment) at t = 0 and dynamics. From Fig. 3a (frames 1, 2) we see that the structure of the projected liquid shell, according to calculation, includes an outer region of reduced density (cavitation zone I) and a narrow inner zone of increased density (liquid layer II). The thickness of the cavitation zone for the times indicated changes little; for spherical symmetry of the flow this corresponds to the spreading of the gas—liquid mixture. Cavitation is not recorded at $t = 53\cdot10^{-6}$ sec (Fig. 3a, frame 3) because of the low density of the mixture in the zone. The velocity of the boundaries of the projected shell is slightly lower in the experiment (~15%) than in the calculations, possibly because of the strength properties of the external hard shell.

A film strip of the dispersal process, including later times, is shown in Fig. 3b for the experimental conditions indicated above. A streamer-type structure with a regular distribution of streams forms on the outer boundary of the shell. Their relative mass evidently is low since they are not seen in the x-ray photographs. The velocity of the streams can be higher than that of the shock wave (SW) in air (Fig. 3b, c), and their formation is due in particular to the displacement of liquid through cracks in the fracturing outer hard shell. A film strip of the process is shown in Fig. 3c for the same loading conditions, but with an outer shell made of thin rubber. The outer boundary in that case remains stable.

Conclusion. Under an explosive load liquid with a free surface breaks up into drops. Let us note some of the distinctive features stemming from the fragmentation of the liquid and possible estimates of typical drop size. In the numerical analysis carried out we disregarded the internal stresses that arise when the cavitating liquid spreads. At the same time, these stresses may arise because of the deformation of bubbles in the velocity field formed (deviation from the symmetry of microflows). This leads to a buildup of bubbles with excess free energy and formation of microstresses in the liquid near the parts of bubbles with the maximum curvature. Moreover, fine-scale effects that are nonequilibrium with respect to pressure and are due to the inertia of the mass of liquid associated with the bubbles occur in the cavitation zone, causing stresses of the type of Reynolds turbulent pulsations [13]. These processes should manifest themselves particularly when the bubble reach the limiting concentrations α_*, corresponding to their close packing ($0.52 < \alpha_* < 0.75$). Clearly, in this case it becomes energetically more advantageous for new free surfaces (breaks) to form in the directions of maximum deformation of the media by the indicated stresses.

The time t_* in which the lowest limiting concentration is reached can be evaluated as $t_* \simeq 1/\lambda$ ($\lambda = \operatorname{div} \mathbf{u}(\xi)$ is the strain rate of the medium in the cavitation zone in the approximation under consideration). This time can be regarded as the characteristic time of the onset of the breakup of the medium into fragments. Since the limiting concentrations α_* are reached unevenly in the entire volume (see Fig. 1c), the fragmentation is a nonuniform process that is stretched out over time.

The characteristic size d of the drops formed can be estimated from energy considerations [14]. Equating the kinetic energy of the deformation motion of a liquid particle of size d to the energy expended on the formation of the free surface, $d^3\rho(d\lambda)^2 \simeq \sigma d^2$ (ρ is the density of the liquid and σ is the surface tension), we obtain $d \simeq (\sigma/\rho\lambda^2)^{1/3}$.

Our calculations have shown that in the range $2 \le m \le 10$ we have $10^3 < \lambda < 10^5$ sec^{-1}, which for water corresponds to the drop sizes in the range $2\cdot10^{-5} \lesssim d \lesssim 50\cdot10^{-5}$ m and characteristic times $10^{-5} \lesssim t_* \lesssim 10^{-3}$ sec. The order of magnitude is entirely feasible.

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